Institute Technical Council

MATHS N PHYSICS IIT BOMBAY

BAZINGA!

MATHS

PRELIMS

DIRECTIONS:

- One by one, 15 questions will be shown on the screen for 90 seconds each.
- After one run-through of all questions, they will be shown again in the same order, for 30 seconds each.
- Write all answers neatly in the sheet provided (with your name and roll number on top).
- For multiple choice questions, just write the option, like Q1. a).
- In the event of a tie, the person who solves more star-marked questions will qualify.

SAFETY SLIDE

1.

This equation is false. Make it true by adding as few operators as possible:

987654321 = 123456789

Only the operators + - × ÷ are allowed.
You can change both sides, but not the numbers.

SAFETY SLIDE

2.

Ted is eating olives with eating speed proportional to the square of the number of uneaten olives. A day after he opened a can, there were 32 of them. A day later, there were just 17 left. How many olives were in the can initially? Assume that the number of olives is a continuous variable for Ted.

SAFETY SLIDE

3.*

Given a cyclic quadrilateral $ABCD$ with

$AB = 4\sqrt{3}, AD = \sqrt{3}, \angle BAC = 30^\circ, \angle CAD = 30^\circ$

find AC

SAFETY SLIDE

4.

This mathematician spent seven secretive years pursuing a dream inspired in childhood. His proof, unveiled in the 1990s, solved a riddle that had defied the greatest minds for three hundred years and earned him worldwide acclaim and knighthood. Who is he?

SAFETY SLIDE

5.

Consider the hourglass figure formed by the lines
$x=y$, $x=-y$, $y=10$, $y=-10$.
If we choose 2 points inside the square formed by the corner of the hourglass, what is the probability that more of the line segment formed by them lies inside the hourglass than outside?

SAFETY SLIDE

6.

The ____ is a concept which describes how patterns in nature, such as stripes and spots, can arise naturally and autonomously from a homogeneous, uniform state.

Fill in the blanks.

SAFETY SLIDE

7.

Let H_n be defined as $H_n = \sum_{k=1}^{n} \frac{1}{k}$.

Find the value of S, where $S = \sum_{n=1}^{\infty} \frac{H_{n+1}}{n(n+1)}$

SAFETY SLIDE

8.

Find the number of positive integers n such that

$$n + 2n^2 + 3n^3 \ldots + 2025n^{2025}$$

is divisible by $(n - 1)$

SAFETY SLIDE

9.

There are n points lying on a circle. Consider the line segments connecting any two of these points. What is the maximum number of unique triangles that can be constructed such that all the vertices of the triangle lie inside the circle (not on the circle)?

SAFETY SLIDE

10.

From a point P outside a circle with center at C, tangents PA and PB are drawn, and they satisfy

$$\frac{1}{CA^2} + \frac{1}{PA^2} = \frac{1}{16}$$

Then the length of chord AB is?

SAFETY SLIDE

11.

A large number of number pairs (x, y) with $0 \leq x, y \leq 1$ are randomly generated (i.e. both numbers in a pair are randomly generated for each pair repeatedly). The distribution of the higher number of each pair resembles which of the following distributions? (Where r is also randomly generated, $0 \leq r \leq 1$)

a) $1/r$

b) $\sqrt{r}$

c) e^{-r}

d) $\cot(r)$

SAFETY SLIDE

12.

A perfect logician has to guess a 3-digit number abc ($a \leq b \leq c$). She is given a positive integer k (only known to her). She has 3 available hints (revealed in order) to guess abc.

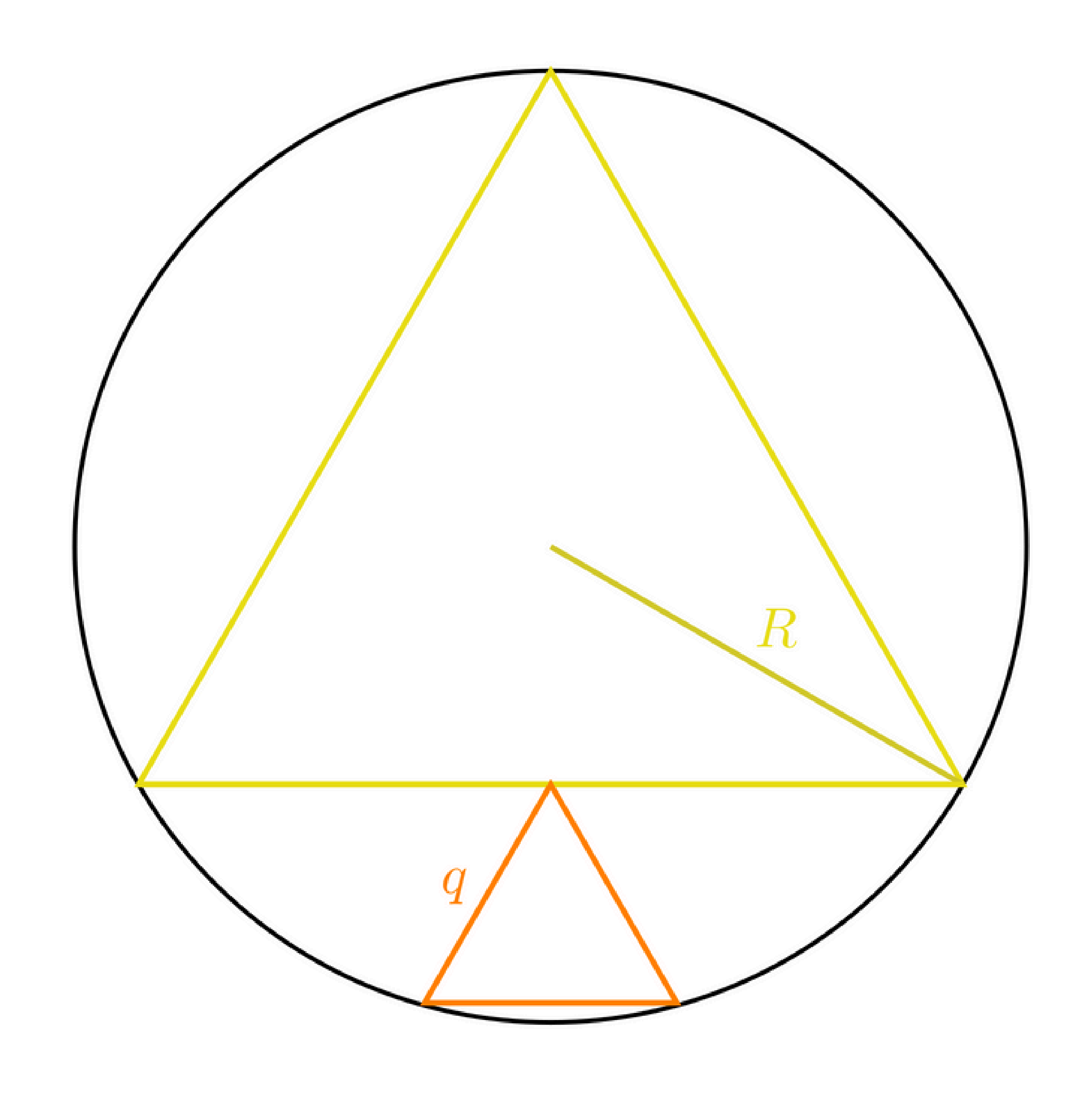
1. $a \times b \times c = 36$
2. $a + b + c = k$
3. $a, b \neq c$

The logician required all 3 hints. What is the number abc?

SAFETY SLIDE

13.

Find q in terms of R. Both are equilateral triangles



SAFETY SLIDE

14.

Based on the image, what could this text translate to?

SAFETY SLIDE

15.*

Nirav recently discovered an interesting gambling game and, unfortunately, his net worth is \$8. In each round he wins \$1 with probability 1/3 and loses \$1 with probability 2/3. He swears he'll quit the moment he's up by \$2. What is the probability that Nirav manages to quit by going up \$2 (as opposed to losing all his money)?

SAFETY SLIDE

2nd VIEWING

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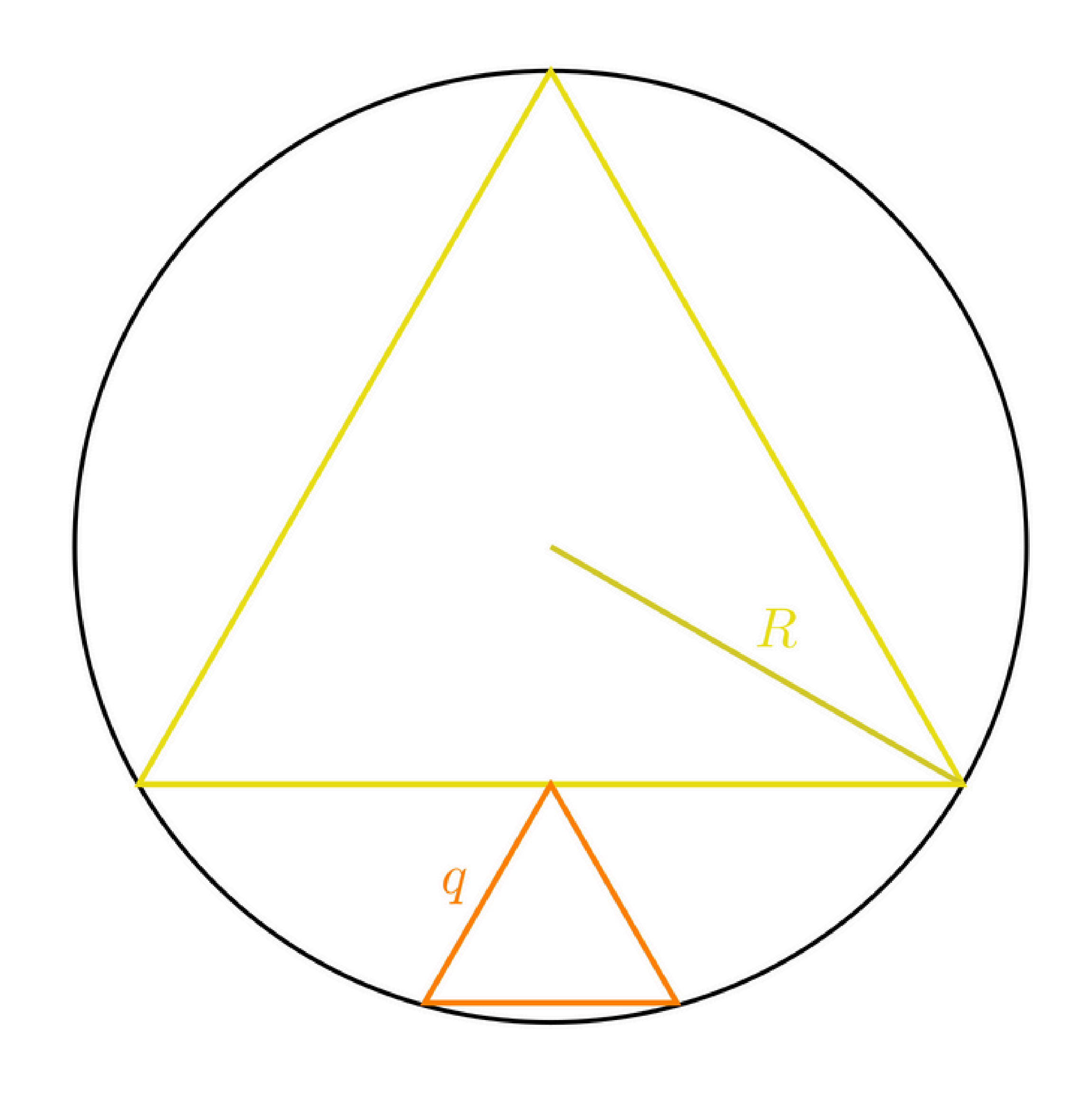

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ANSWERS

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9876-5432+1 = 1-2345+6789

or

9-8765+4321 = 1234-5678+9

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272

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5

(using Ptolemy's)

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Andrew Wiles

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1/2

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Fill in the blanks.

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Turing Pattern

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2

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Find the number of positive integers n such that

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Number of divisors of (2025*2026)/2 =

30

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$$^{n}C_{6}$$

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d) $\cot(r)$

b) √r

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229

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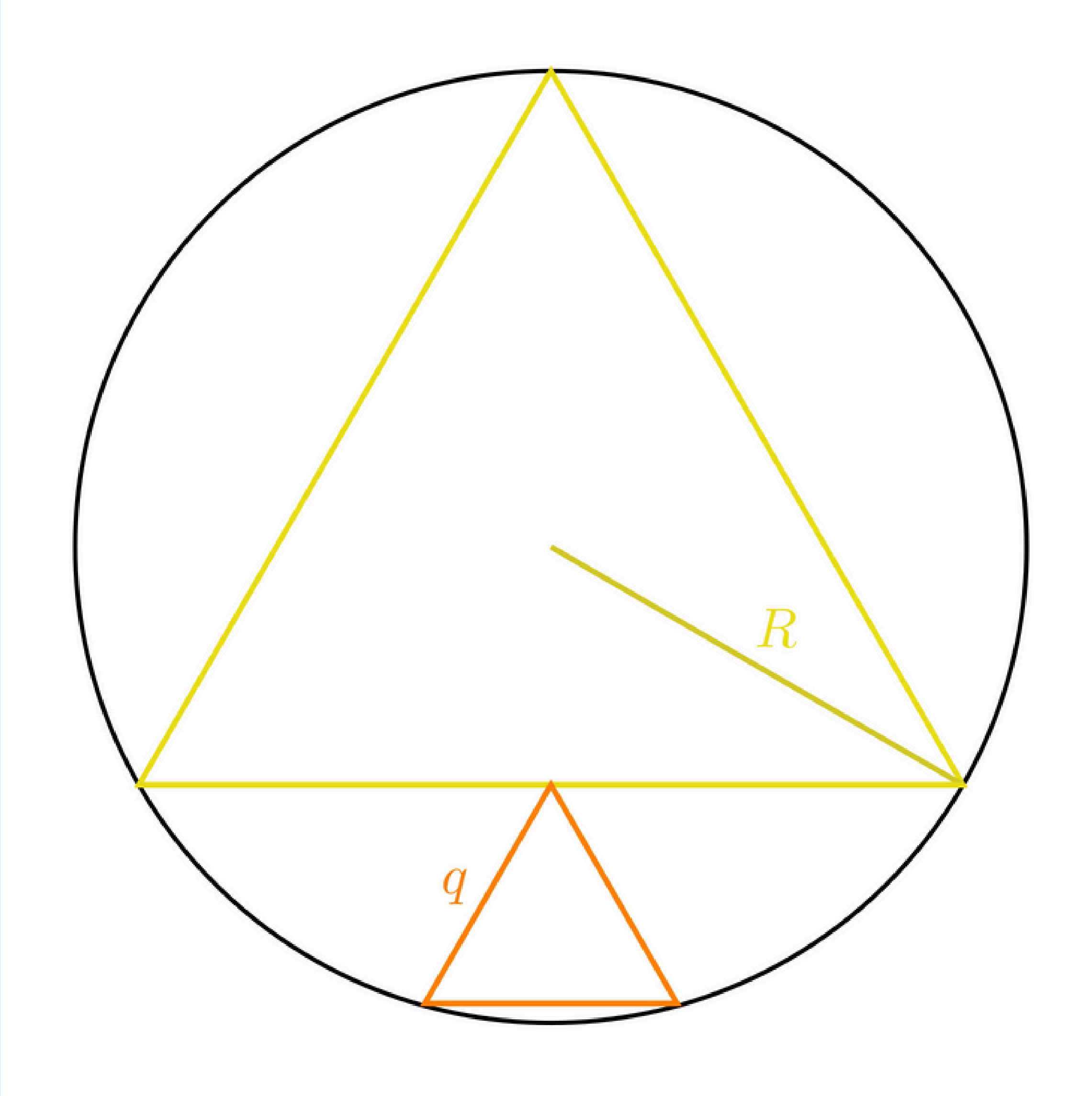
Find q in terms of R. Both are equilateral triangles



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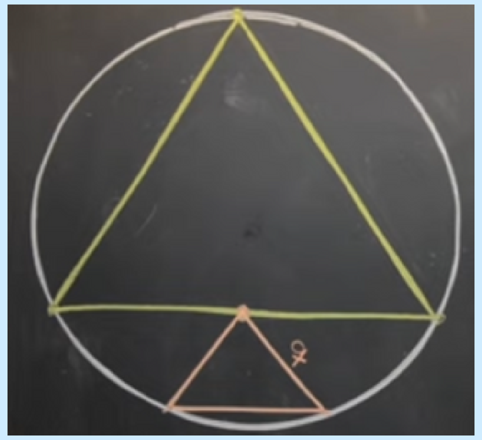

$(\sqrt{15} - \sqrt{3})R/4$

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Archimedes -

Don’t disturb my circles

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85/341

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